STABLE DECONVOLUTION OF NOISY LIDAR SIGNALS

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ABSTRACT

A main application of lidar remote sensing is to provide spatial resolved data. Based on the fundamental relationship between space and time the distance can be calculated from the photons' time of flight. Accordingly, the distance resolution is limited by the time resolution of the lidar detector. Furthermore, if the system response function of the lidar is longer than the time resolution interval of the detector, the measured lidar signal is smeared, and the effective distance resolution decreases. In theory, this loss of resolution can be corrected by deconvolution of the measured signal with the system response function. Measured lidar signals are superposed by noise which makes a direct deconvolution impossible because of the effect of noise amplification.

In this paper, a technique is presented which allows for a stable deconvolution of lidar signal returns without any filtering in the frequency domain. It is based on the Richardson-Lucy algorithm for image reconstruction. Simulations of short distance lidar signals have been used to compare the method with conventional deconvolution algorithms such as the Fourier transformation.

INTRODUCTION

Time-resolved lidar signals P(t) are normally analysed by the well-known single scattering lidar equation.[1] The relation between time t and distance x is given by

$$x = \frac{c \cdot t}{2} \;, \qquad (1)$$

where c is the speed of light in the corresponding medium. The distance resolution Dx is therefore determined by the time resolution Dt of the detector and the length and shape of the laser pulse. If the system response function $R(t)$ of the lidar, often denoted as the system function, is much longer than the sampling interval Δt, the distance resolution Dx increases. This is mathematically described by the convolution integral

$$P(t) = \int_{-\infty}^{\infty} R(t'-t) P_d(t')\,dt' \equiv R(t) \times P_d(t) \quad . \qquad (2)$$

In this equation, $P_d(t)$ is a signal that could be measured with an infinitively short detector response time and laser pulse length. $P_d(t)$ can be called the impulse response function of the environment. It reflects the true distribution of optically active substances along the ray path of the lidar.

METHODS

Fourier transformation

Assuming a known system response function $R(t)$ of the instrument, the deconvolution of the lidar signal $P(t)$ by $R(t)$ improves the distance resolution. L. Gurdev et al. developed deconvolution techniques that are based on Fourier transforms and on the solution of the Volterra integral equation.[2] The convolution theorem is

$$P(t) = R(t) \times P_d(t) \quad \Leftrightarrow \quad \mathrm{F}P(f) = \mathrm{F}R(f) \cdot \mathrm{F}P_d(f) \quad . \qquad (3)$$

´F´ stands for Fourier transform, ´F^{-1}´ for its inversion. This leads to the deconvolution equation

$$P_d(t) = \mathrm{F}^{-1}\left(\frac{FP(f)}{FR(f)}\right) \quad . \tag{4}$$

Lidar data are normally digitised so that Eq. (2) can be expressed in a discrete form. $P(t_n)$ and $P_d(t_n)$ are connected by the convolution matrix M:

$$P(t_n) = M \cdot P_d(t_n) \tag{5}$$

with

$$M = \begin{pmatrix} R(t_1) & 0 & 0 & \cdots & 0 & 0 \\ R(t_2) & R(t_1) & 0 & 0 & \cdots & 0 \\ \cdots & R(t_2) & R(t_1) & 0 & \cdots & 0 \\ R(t_m) & \cdots & R(t_2) & R(t_1) & 0 & \vdots \\ 0 & R(t_m) & \cdots & R(t_2) & R(t_1) & \ddots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} .$$

Y. Je Park et al. calculated the inverse convolution matrix M^{-1} in order to perform a deconvolution by matrix multiplication:[3]

$$P_d(t_n) = M^{-1} \cdot P(t_n) \tag{6}$$

In real measurements, lidar signals are always superposed by noise $N(t)$:

$$P_m(t) = P(t) + N(t) = \left(R(t) \times P_d(t)\right) + N(t) \quad , \tag{7}$$

and this additive term makes a direct deconvolution impossible. In cases, however, where the functional shape of the undisturbed signal $P(t)$ is known, e. g. in fluorescence decay time measurements, the number of free parameters is reduced significantly, and Eq. 7 can be numerically solved by a least squares fit.

Lidar profiles are strongly dependent on the optical geometry of the instrument and on the composition and spatial distribution of substances in the medium. The system function $R(t)$ is mainly governed by the laser pulse shape, and by the photomultiplier and digitiser response function. In general, the convolution with $R(t)$ shows a low pass characteristic. A deconvolution with Eq. 4 or 6 therefore intensifies the range of higher frequencies, where noise contributions to the signal are relevant. In consequence, simple low-pass filtering can be very useful in some cases. The cut-off frequency can be derived from the power spectrum of $R(t)$, and the window length should be adapted to the length of the pulse response. However, low-pass filtering of the measured signal $P_m(t)$ will not only affect noise but also the undisturbed lidar signal $P(t)$. An optimal filter is the Wiener filter $F(f)$, defined by

$$\mathrm{F}P_w(f) = \frac{\mathrm{F}P_m(f) \cdot F(f)}{\mathrm{F}R(f)} \tag{8}$$

so that

$$\left\| P_w(t) - P_d(t) \right\|_2 = \int_t \left(P_w(t) - P_d(t)\right)^2 = \min \quad . \tag{9}$$

The Wiener filter[4] can be constructed by

$$F(f) = \frac{|\mathrm{F}P(f)|^2}{|\mathrm{F}P(f)|^2 + |\mathrm{F}N(f)|^2} \quad . \tag{10}$$

Eq. 10 requires knowledge - or at least a hypothesis - of the power spectral density of the noise and the undisturbed signal. There is no way to derive this from the measured signal $P_m(t)$ alone. Press et al. proposed to select a cut-off frequency ´by eye´ from the power spectral density of the measured signal $|\mathrm{F}P_m(f)|^2$.

Non-negative least squares algorithm.
Combining Eqs. 5 and 7 yields

$$P_m(t_n) = P(t_n) + N(t_n) = M \cdot P_d(t_n) + N(t_n) \tag{11}$$

from which follows $M \cdot P_d(t_n) - P_m(t) = -N(t_n)$,
hence:

$$\begin{aligned} \|M \cdot P_d(t_n) - P_m(t_n)\|_2 &= \frac{1}{n_{\max}} \sum_n \left(M \cdot P_d(t_n) - P_m(t_n)\right)^2 \\ &= \frac{1}{n_{\max}} \sum_n N(t_n)^2 = \|N(t_n)\|_2 \end{aligned} \quad . \tag{12}$$

Without any prior information about $P_d(t)$ and $N(t)$ we may assume that the noise is minimized. $P_d(t)$ is the signal which would be received by an ideal lidar. It holds $P_d(t) \geq 0$ at any time t. The deconvolution problem with respect to $P_d(t)$ can be expressed in the form

$$\|M \cdot P_d(t_n) - P_m(t_n)\|_2 = \min \ , \quad P_d(t_n) \geq 0 \forall t_n \tag{13}$$

Lawson and Hanson developed a method called ´non-negative least squares´ (NNLS) algorithm to solve this general problem (see appendix).[5]

Richardson-Lucy algorithm.
The Richardson-Lucy algorithm is an iterative algorithm which was developed for image restoration early in the 1970´s independently by W. H. Richardson and L. B. Lucy. [6,7] It is derived directly from Bayes theorem. When we regard a lidar profile as an image with the dimension 1 x N, the Richardson-Lucy algorithm can be used for deconvolution. Eq. 14 shows its adapted form. The i-th iteration can be calculated by

$$P_d^{(i+1)}(t) = P_d^{(i)}(t) \cdot \left(R^T(t) \times \frac{P_m(t)}{P_m^{(i)}(t)} \right) \tag{14}$$

with $P_m^{(i)}(t) = R(t) \times P_d^{(i)}(t)$.

The measured signal $P_m(t)$ is taken as the initial guess $P_d^{(1)}$ for the iteration. Negative values due to noise have to be set to zero in order to guarantee its convergence. It is easy to see from Eq. 14 that the total signal power will be conserved.

SIMULATION

A simulated lidar signal has been used to test the feasibility of range resolution improvement by deconvolution. The lidar parameters correspond to data for an underwater lidar;[8] the sampling rate is increased from 250 Mhz (*Dt*=2 ns) to 500 Mhz (*Dt*=1 ns). Natural decay times and multiple scattering effects are neglected.

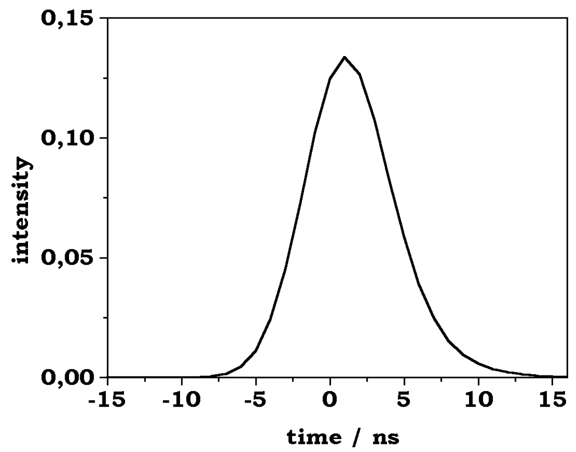

Figure 1: Normalised system response function R(t)

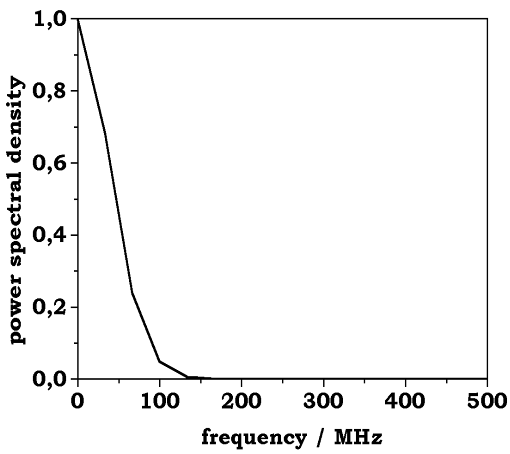

Figure 2: Power spectral density $|FR(f)|^2$

The system response function of the lidar can be regarded as a convolution of laser pulse shape and photomultiplier decay function. The laser pulse has a Gaussian shape with a standard deviation of 2.5 ns. The photomultiplier impulse response shows an exponential decay with a time constant of 2 ns. Figure 1 shows the normalised lidar system function *R(t).* The shape of *R(t)* is identical with the lidar signal $P_m(t)$ which originates from a scattering δ-peak in a non-absorbing and non-scattering medium. For real measurements, *R(t)* will be derived in this way. The influence of fluctuations in *R(t)* on the deconvolution process has been investigated by Dreischuh et al..[9] The power spectral density $|FR(f)|^2$ of the lidar system function is plotted in Figure 2. Its low-pass characteristic is obvious. A low-pass filter with a cut-off frequency at 200 Mhz and a window length of 32 ns is reasonable to avoid noise intensifying. Application of this filter will limit the maximum distance resolution to $\Delta R = 0.5 \cdot c_{water} \cdot 5$ ns ≈ 0.56 m.

Figure 3 draws the virtual lidar signal $P_d(t)$. In general, lidar signals show a more dynamic behaviour with strongly decreasing intensity over distance because of the geometrical form factor and beam attenuation. However, as the distribution of the measured substance within the water column is freely variable, the functional shape of a lidar signal is not determined.

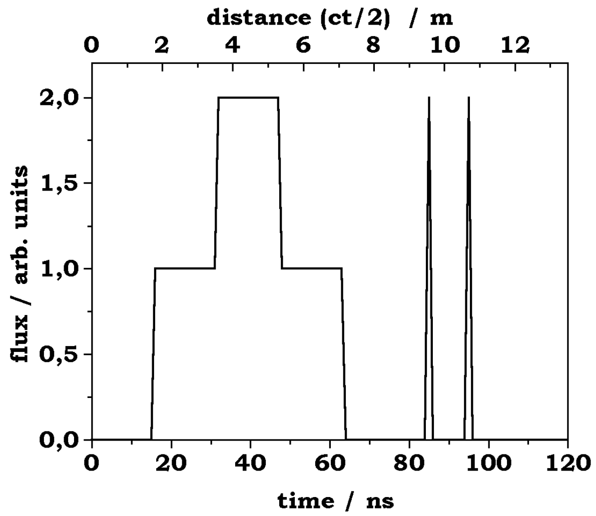

Figure 3: Ideal lidar signal $P_d(t)$

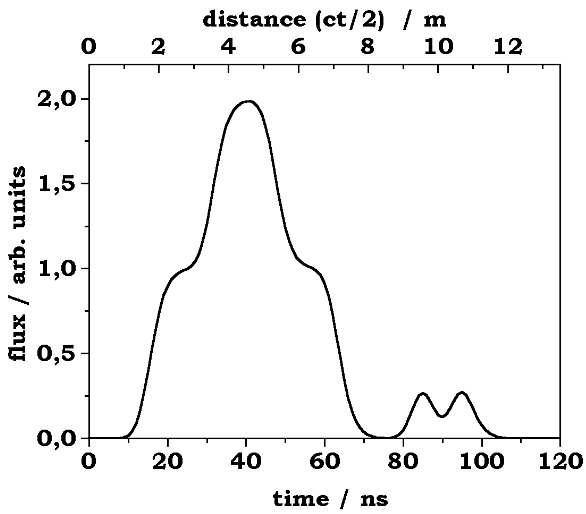

Figure 4: Simulated lidar signal P(t) (Eq. 2)

The profile consists of three plateaus between 16 ns and 64 ns and two δ-peaks in 85 ns and in 95 ns distance. Each of the algorithms which are described above lead to an exact solution of Eq. 2 when noise is absent. $P_d(t)$ can be calculated directly from *P(t)* using *R(t).* In order to test the stability of the deconvolution algorithms, ´real´ signals $P_m(t)$ have been produced by adding an artificial noise to *P(t)* (Eq. 7). The noise is a random Gaussian distribution and is typically defined as shot noise; its standard deviation is 0.1 % of the maximum of the measured signal.

RESULTS

Figures 5 to 10 show the results of a direct deconvolution of the simulated lidar signals with the given system response function. The result of a direct deconvolution by Fourier transformation is plotted in Figure 5. The effect of noise amplification especially in the region of high frequencies can be observed. It can be reduced by prior low pass filtering, Figure 6. Figure 7 shows the result of application of the NNLS algorithm. Peaks are well reconstructed, but the whole signal is overlayed by an amplified noise. Previous low pass filtering improves again the signal quality, Figure 8.

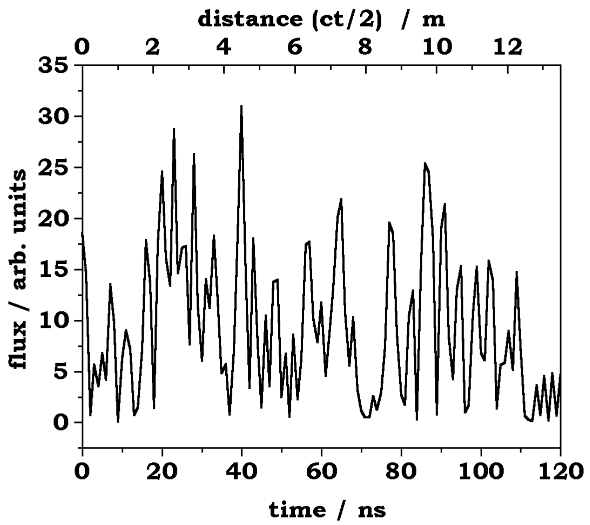

Figure 5: Lidar signal with 0.1 % noise, deconvoluted by Fourier transformation without prior low pass filtering.

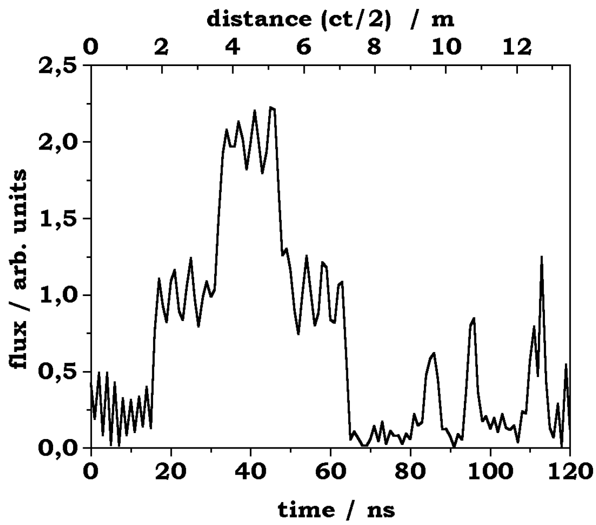

Figure 6: Lidar signal with 0.1 % noise, deconvoluted by NNLS approximation without prior low pass filtering.

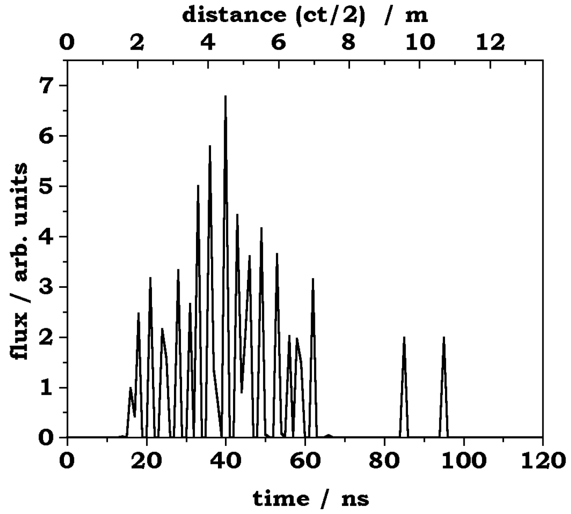

Figure 7: Lidar signal with 0.1 % noise, deconvoluted by Fourier transformation after low pass filtering.

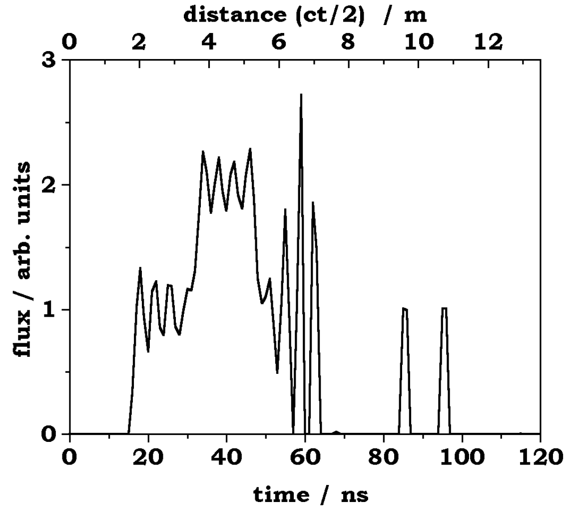

Figure 8: lidar signal with 0.1 % noise, deconvoluted by NNLS approximation after low pass filtering.

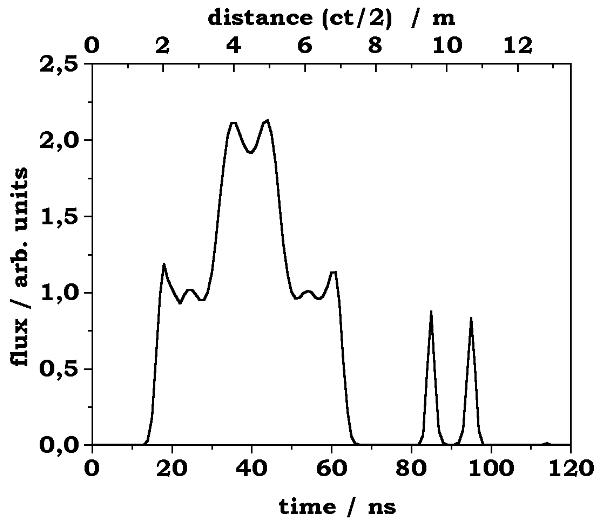

Figure 9: Lidar signal with 0.1 % noise, deconvoluted by the Richardson-Lucy method after low pass filtering (100 iteration steps).

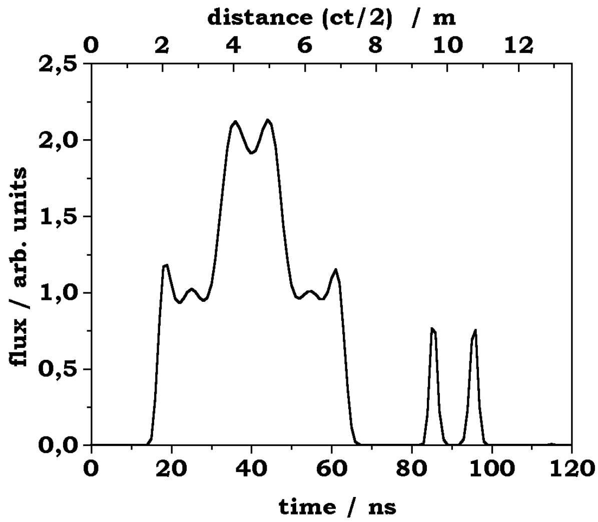

Figure 10: Lidar signal with 0.1 % noise, deconvoluted by the Richardson-Lucy method without prior low pass filtering (100 iteration steps).

Application of the Richardson-Lucy algorithm leads to the data in Figures 9 and 10. The reconstruction of the undisturbed signal (Figure 3) from the measured signal (Figure 4) by the Richardson-Lucy algorithm is better than by Fourier transformation or NNLS. Noise amplification does not appear. The result of the deconvolution is independent of previous low pass filtering.

CONCLUSIONS

The distance resolution of a lidar can be improved by the use of deconvolution techniques. Conventional algorithms which are based on Fourier transforms show a very sensitive behaviour in the presence of noise and therefore require noise removing filters to be applied. The quality of the non-negative least squares deconvolution varies with the shape of the profile. It can restore δ-peaks, but showed to be very unstable when plateaus are to be restored. For the examined lidar profile the 1-dimensional Richardson-Lucy algorithm leads to the best results. The retrieval of the optimum iteration number as well as its perfomance with an increasing noise level has to be investigated in more detail in order to make it a useful tool for lidar data processing.

APPENDIX: NON-NEGATIVE LEAST SQUARES ALGORITHM

The algorithm is reproduced from Lawson and Hanson, 1974.[5]

Let E be an $m_2 \times n$ matrix and f an m_2-vector.

Problem: Minimize $\|Ex - f\|$ with respect to $x \geq 0$

Step	Description
1	set $P := \varnothing$, $Z := \{1,2,...,n\}$, $x := 0$
2	calculate $w := E^T (f - Ex)$
3	if $Z = \varnothing$ or $w_j \leq 0 \;\; \forall j \in Z$, goto step 12
4	find index $t \in Z$, so that $w_t = \max\{w_j ; j \in Z\}$
5	move index t from Z to P
6	define E_P as $m_2 \times n$-matrix by $column\ j\ of\ E_P := \begin{cases} column\ j\ of\ E,\ if\ j \in P \\ 0,\ if\ j \in Z \end{cases}$
	calculate the n-vector z as a solution of the least squares problem $E_P z \cong f$
	notice that only the components $z_j, j \in P$ are determined
	define $z_j := 0 \; \forall j \in Z$
7	if $z_j > 0 \; \forall j \in P$, set $x := z$ and goto step 2
8	find the index $q \in P$ so that
	$x_q / (x_q - z_q) = \min \{x_j / (x_j - z_j) : z_j \leq 0, j \in P\}$
9	set $a := x_q / (x_q - z_q)$
10	set $x := x + a(z - x)$
11	move form P to Z all indices $j \in P$ with $x_j = 0$
	goto step 6
12	end of calculation

A proof of the finite convergence of the algorithm is also given in Lawson and Hanson, 1974.[5]

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